Math 62: 8.3 Piecewise Functions
Lesson #2

Math 72: 3.4 Piecewise Functions
Lesson # 11-12

3.8 -2nd Piecewise Functions

Objectives: 1) Graph piecewise-defined functions.

To do this we need several smaller objectives:

a) Understand notation used for piecewise functions.

b) Evaluate piecewise functions

c) Determine y-values at endpoints of piecewise functions.

d) Determine if endpoints should be graphed with ● or O.

GC 21: Graphing Piecewise Functions

M70 3.8 – 2nd

An example of a piecewise-defined function, also called a piecewise function:

$$f(x) = \begin{cases} 2x+3 & \text{if } x \le 0 \\ -x-1 & \text{if } x > 0 \end{cases}$$

Note: The word "if" is optional

$f(x)$ is the name of this function.

The brace { tells us it's piecewise.

$(2x+3)$ and $(-x-1)$ are the two pieces.

$x \le 0$ is the domain for the first piece $(2x+3)$.

$x > 0$ is the domain for the second piece $(-x-1)$

Note: It has one name because it's all one function. The resulting graph MUST PASS THE VERTICAL LINE TEST!

1. Using $f(x)$ above, evaluate.
   a) $f(3)$
   b) $f(1)$
   c) $f(0)$
   d) $f(-1)$

① continued.

a) $f(3)$

step 1: Identify which domain is true for this x.

$x=3$ is greater than 0 $\Rightarrow$ $x>0$.

step 2: Use the piece which applies to that domain. (Ignore the other piece!)

$-x-1$ if $x>0$.

step 3: Evaluate the appropriate piece.

$$-x-1$$
$$= -(3)-1$$
$$= -4$$

$\boxed{f(3) = -4}$

b) $f(1)$

step 1: (as above) $x=1$ is $x>0$.

step 2: $-x-1$ if $x>0$

step 3: $-(1)-1$

$\boxed{f(1) = -2}$

c) $f(0)$

step 1: $x=0$ is $x \le 0$

step 2: $2x+3$ if $x \le 0$

step 3: $2(0)+3$

$\boxed{f(0) = 3}$

d) $f(-1)$

step 1: $x=-1$ is $x \le 0$

step 2: $2x+3$ if $x \le 0$

step 3: $2(-1)+3$

$\boxed{f(-1) = 3}$

M70

(2) Using the same f(x), evaluate to complete the tables.

Hint: Use GC table function.

x	f(x)
−5	
−4	
−3	
−2	
−1	
0	

x	f(x)
3	
2	
1	
1/2	
1/4	
1/8	

Solution:

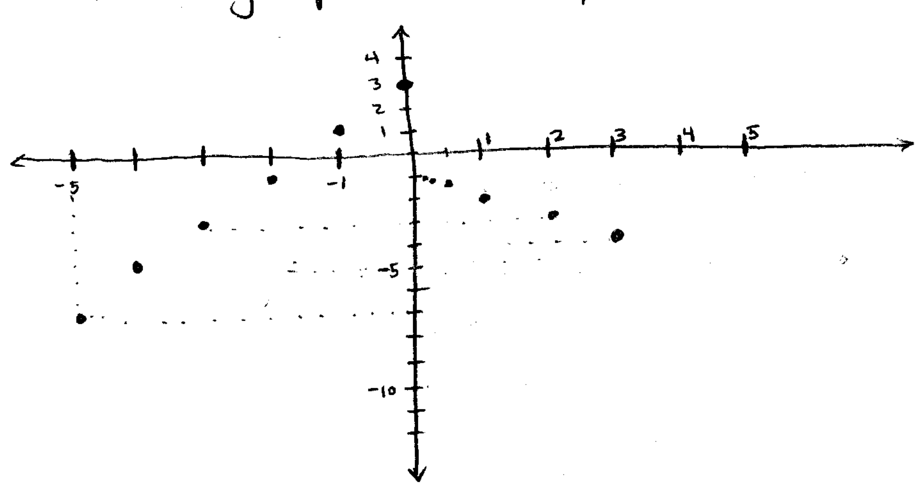
x	f(x)
−5	−7
−4	−5
−3	−3
−2	−1
−1	1
0	3

use 2x+3 piece

x	f(x)
3	−4
2	−3
1	−2
1/2	−1.5
1/4	−1.25
1/8	−1.125

use −x−1 piece

(3) Sketch the graph of the points we know so far.



④ Complete the graph by
- calculating endpoints of each piece
- using ● to graph piece where endpoint is included ($x \le 0$ includes $x=0$).
- using ○ to graph piece where endpoint is not included ($x > 0$ does not include $x=0$).
- connecting dots within each piece

The endpoints occur when $x=0$, from domains.

$f(0)=3$ is the ● endpoint of the piece $2x+3$.

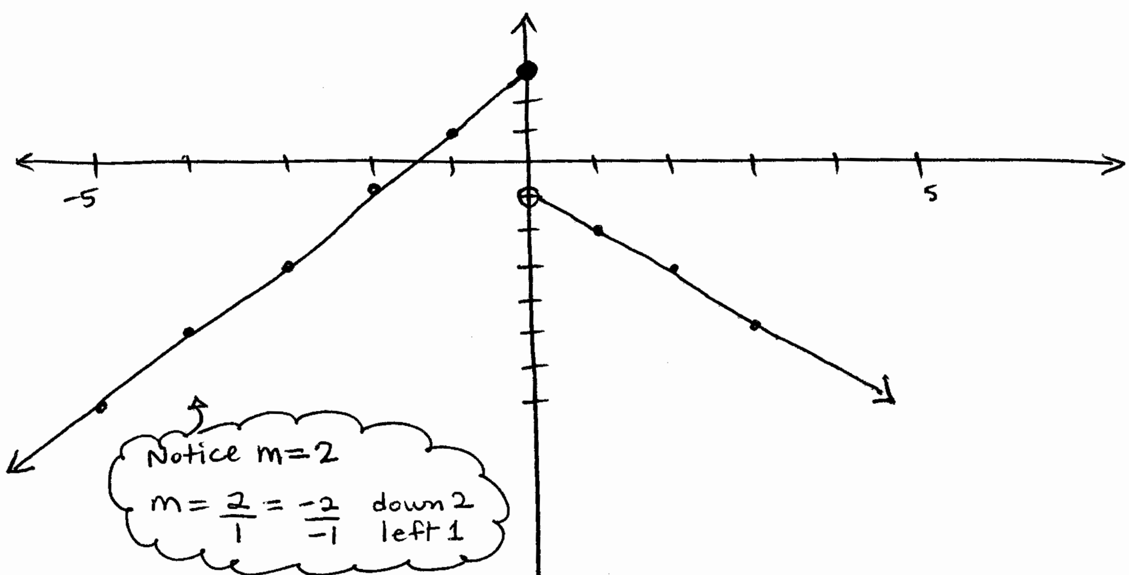
The endpoint of the piece $-x-1$ is also when $x=0$.
But this endpoint is not included.
Substitute $x=0$ to get y-coordinate.

$-0-1=-1$

Graph $(0,-1)$ using open circle ○.

Connect all the $x \le 0$ pts together.
Connect all the $x > 0$ pts together.

<u>Note</u>: The left piece may or may not connect to the right piece.



<u>Note</u>: Do NOT put arrowheads at the endpoints where domains are divided.

<u>Note</u>: Do put arrowheads on the ends where $x \to +\infty$ or $x \to -\infty$.

(5) Sketch graph of $f(x) = \begin{cases} -2x+4 & x \le -1 \\ 3 & x > -1 \end{cases}$

step 1: $-2x+4$ is a line with slope -2

3 is a horizontal line

step 2: Endpoints occur when $x = -1$

$f(-1) = -2(-1)+4 = 6$

plot $(-1, 6)$ using • included endpt.

$3 \Rightarrow$ plot $(-1, 3)$ using o excluded endpt.

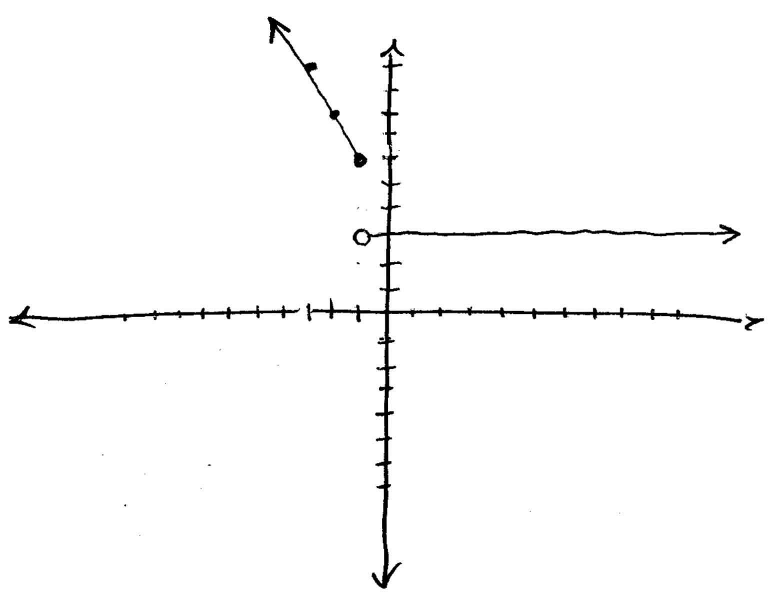
step 3: To graph $-2x+4$, $x \le -1$

plot $(-1, 6)$, use $m = \frac{-2}{1} = \frac{2}{-1} = \frac{2 \text{ up}}{1 \text{ left}}$

using ●

To graph 3, $x > -1$

plot $(-1, 3)$ using o, draw horizontal.



Yes, it passes the V.L.T!

Helpful Hint: For the $x < \#$ or $x \le \#$ piece, write slope as fraction with negative denominator. (go left).

For the $x > \#$ or $x \ge \#$ piece, write slope as fraction with positive denominator (go right)

Procedure for graphing piecewise functions with linear pieces.

$$f(x) = \begin{cases} ax + b & x \le c \\ dx + e & x > c \end{cases}$$

1) draw axes.
2) identify $x=c$ on graph, left & right pieces
$x \le c$ $\quad x > c$
(or $x < c$ & $x \ge c$)

3) Subst $x=c$ into 1st piece
use ● if $x \le c$ or $x \ge c$
use o if $x < c$ or $x > c$

4) use slope of first piece to fill in line in correct direction
- check domain
- check sign of slope

5) repeat steps 3 & 4 for 2nd piece.

6) check that final graph passes VLT.

Now that we have explored a problem in detail, let's write our method.

<u>Step 1</u>: Identify the shape of each piece. [In M70, usually lines.]

<u>Step 2</u>: Evaluate endpoints in both pieces. Determine which is ● and which is ○ on graph.

<u>Step 3</u>: Graph each piece.

<u>Step 4</u>: Check that final graph passes V.L.T.

⑤ Sketch graph of $f(x) = \begin{cases} x+2 & \text{if } x<1 \\ 2x+1 & \text{if } x \geq 1 \end{cases}$

<u>Step 1</u>: $x+2$ is a line $y=x+2$ with slope 1.
$2x+1$ is a line $y=2x+1$ with slope 2.

<u>Step 2</u>: Endpoints occur when $x=1$.

$f(1) = 2(1)+1 = 3$

plot $(1,3)$ using ● included endpt

In other piece $1+2=3$

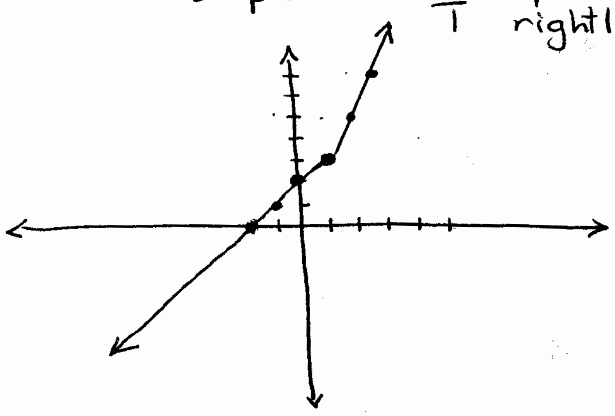
plot $(1,3)$ using ○ excluded endpt

Note: In this graph, the endpoints overlap!

<u>Step 3</u>: To graph $x+2$, $x<1$
plot $(1,3)$ ... with ○ excluded endpoint
use slope $m = \frac{1}{1} = \frac{-1}{-1}$ down 1 left 1.

- - - - - - - - - - - - - - - - -

To graph $2x+1$, $x \geq 1$
Plot $(1,3)$ with ● included endpoint
use slope $m = \frac{2}{1}$ up 2 right 1



Yes! It passes the VLT!

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Extra Practice: Sketch graphs

(7) $f(x) = \begin{cases} x-1 & x \le 3 \\ -x+5 & x > 3 \end{cases}$

(8) $f(x) = \begin{cases} 4x-4 & x < 2 \\ -x+1 & x \geq 2 \end{cases}$

(9) $g(x) = \begin{cases} -3x & x \le -2 \\ 3x+2 & x > -2 \end{cases}$

(10) $k(x) = \begin{cases} -1 & x < -3 \\ 2 & x \ge -3 \end{cases}$

(11) Challenge question

$m(x) = \begin{cases} -x+2 & x < -1 \\ -x^2+4 & x \ge -1 \end{cases}$

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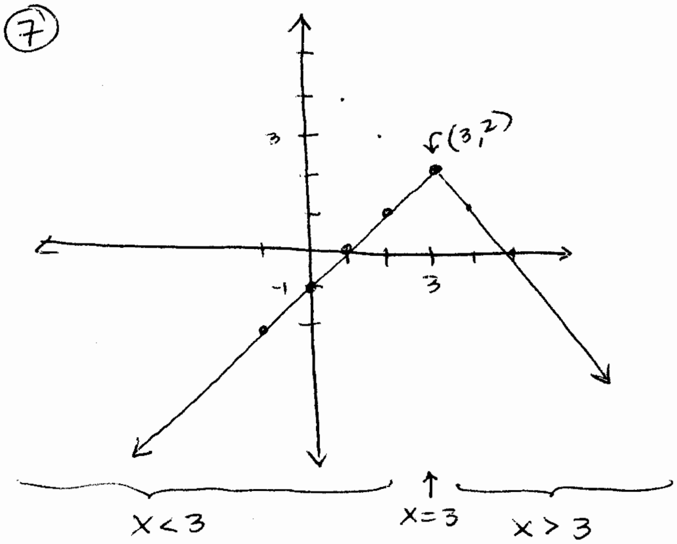
Solutions to Extra Practice

(7)



Note:

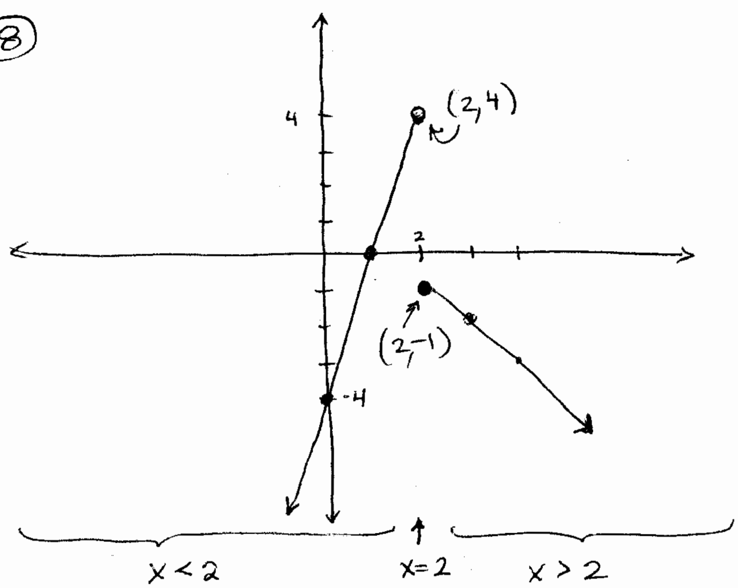
$$f(x) = \begin{cases} x-1 & x \le 3 \\ -x+5 & x > 3 \end{cases}$$

is the same as

$$f(x) = -|x-3| + 2.$$

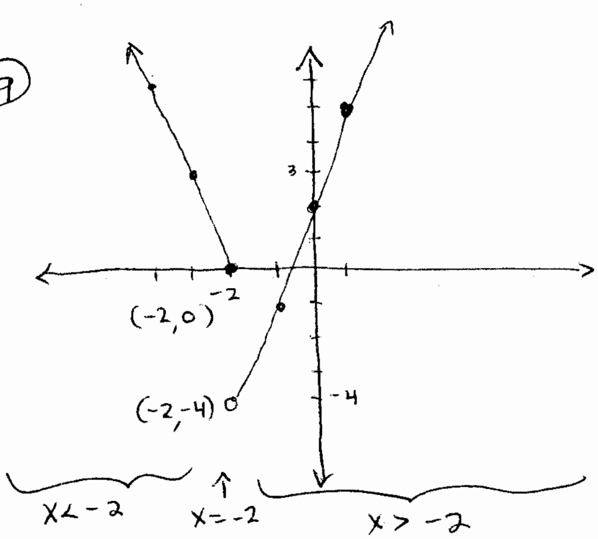
ALL absolute functions can be written as piecewise functions.

(8)



$$f(x) = \begin{cases} 4x-4 & x<2 \\ -x+1 & x \ge 2 \end{cases}$$

(9)



$$g(x) = \begin{cases} -3x & x \le -2 \\ 3x+2 & x > -2 \end{cases}$$

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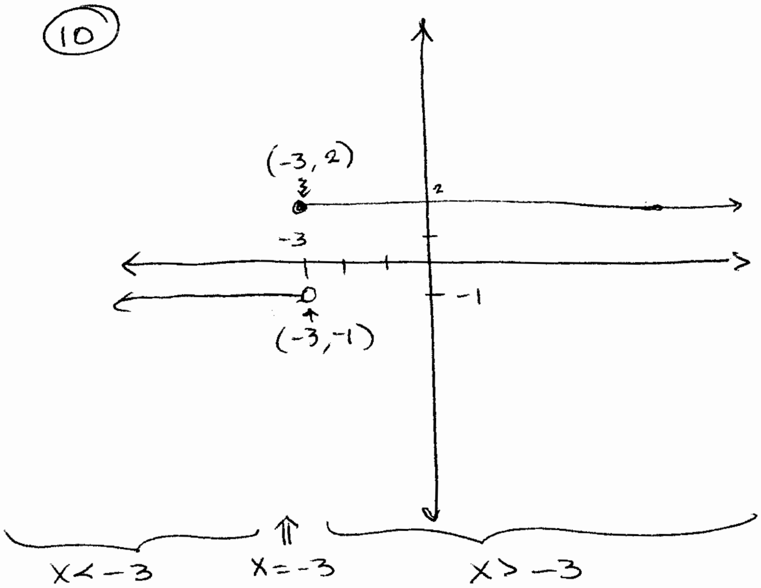

$$k(x) = \begin{cases} -1 & x < -3 \\ 2 & x \geq -3 \end{cases}$$

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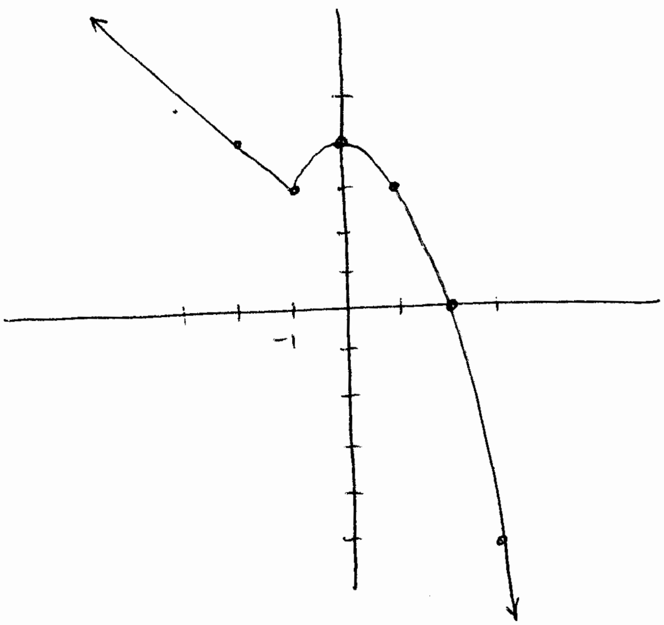
$$m(x) = \begin{cases} -x+2 & x < -1 \\ -x^2+4 & x \geq -1 \end{cases}$$

lines

parabola opening down
vertex at (0,4)



$m(-1) = -(-1)^2 + 4 = 3$

(-1, 3) plotted ●

other piece $-(-1)+2 = 3$

they connect at (-1, 3).

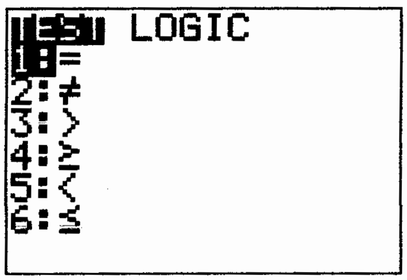
Name ______________________________

Date ______________________________

TI-84+ GC 21 Graphing Piecewise Functions

Objectives: Use TEST to compare a single value of a variable to a test value
Use TEST to compare many values of a variable to a test value
Multiply a function by a TEST result to limit domain and graph a piece
Determine if your GC will show $y = 0$ when multiplied, use divide by a TEST result
Add pieces to graph a piecewise function

The TEST menu is the second function above MATH:



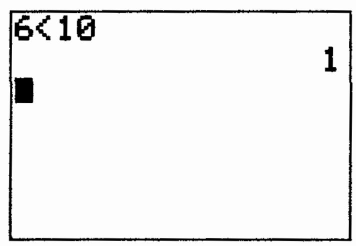
We can select from the list by typing the corresponding number. "Less than" is option 5, for example.

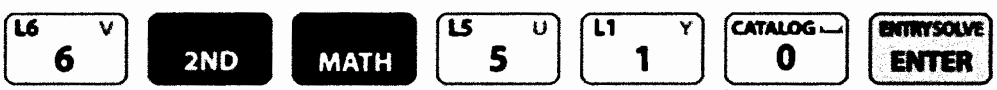
Example 1: Use GC to determine if

a) $6 < 10$

b) $6 \le 3$

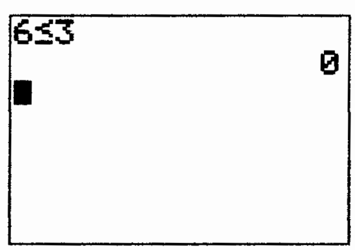
a) Use the TEST menu to determine if $6 < 10$

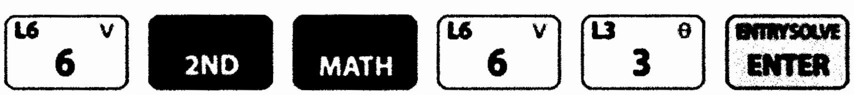




The GC says 1 if the answer to the question is true (or yes).
$6 < 10$ is a true statement, so GC says 1.

b) Use the TEST menu to determine if $6 \ge 3$





The GC says 0 if the answer to the question is false (or no).
$6 \ge 3$ is a false statement, so GC says 0.

Example 2: Use GC to store $x = -1$ in memory and determine if

a) $x < 10$

b) $x \geq 3$

Store $x = -1$ in memory location x (graphing x).

(−) 1 STO> X,T,θ,n ENTER

```
-1→X
                 -1
```

a) Test $x < 10$

X,T,θ,n 2ND MATH 5 1 0 ENTER

```
-1→X
                 -1
X<10
                  1
```

The GC tests the value stored in memory, and concludes that yes, $-1 < 10$.

b) Test $x \geq 3$

X,T,θ,n 2ND MATH 4 3 ENTER

```
-1→X
                 -1
X<10
                  1
X≥3
                  0
```

The GC tests the value stored in memory, and concludes that no, $-1 \geq 3$ is a false statement.

Example 3: Graph the TEST results for

a) $x \leq 4$

b) $x > -2$

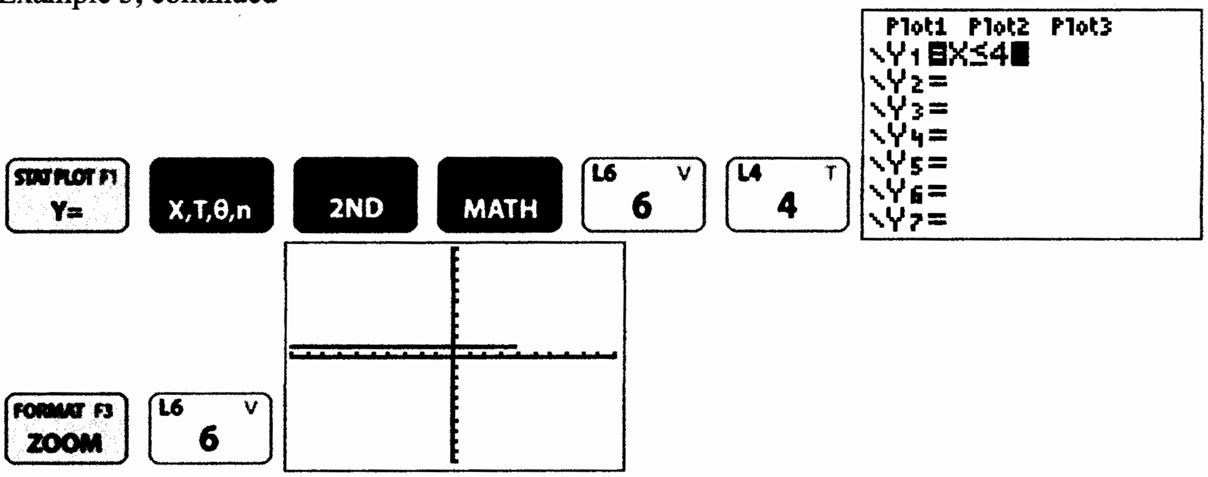
CAUTION: We are NOT graphing the inequality $x \leq 4$, which would be graphed on a number line.

TEST gives y-coordinate that is 0 or 1, depending on whether x is less than or equal to 4 (y-coordinate 1) or not less than or equal to 4 (y-coordinate 0).

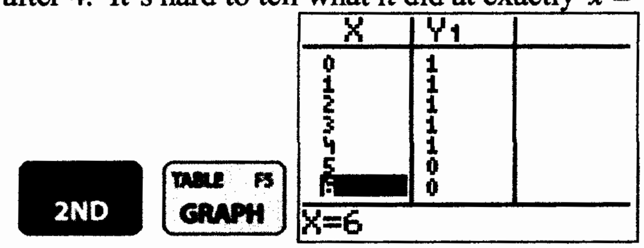
The GC automatically generates the values of x from Xmin and Xmax in WINDOW.

a) Type the test $x \leq 4$ as a function in the Y= menu, then graph in a standard viewing window.

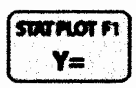
Example 3, continued

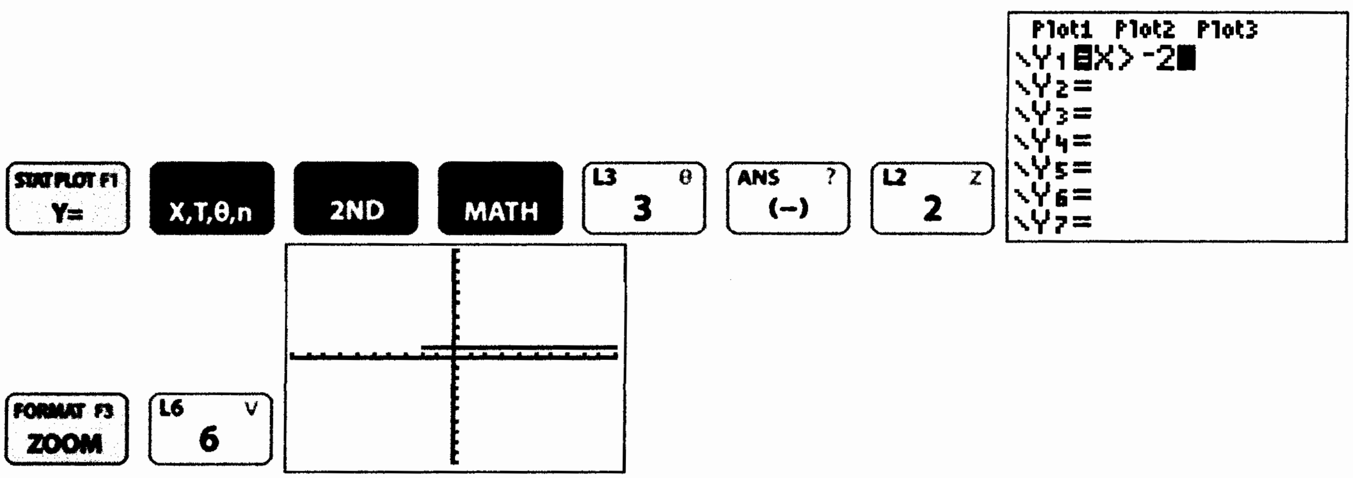


The GC graphs a horizontal line $y=1$ for all x values up to $x=4$, then a horizontal line $y=0$ for all x-values after 4. It's hard to tell what it did at exactly $x=4$ without checking a table.

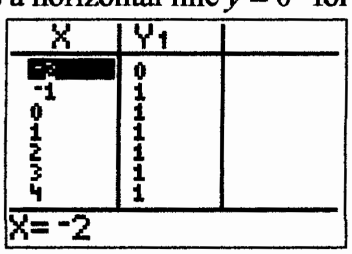


$x=4$ has y-value 1.

b) Type the test $x>-2$ as a function in the  menu, then graph in a standard viewing window.



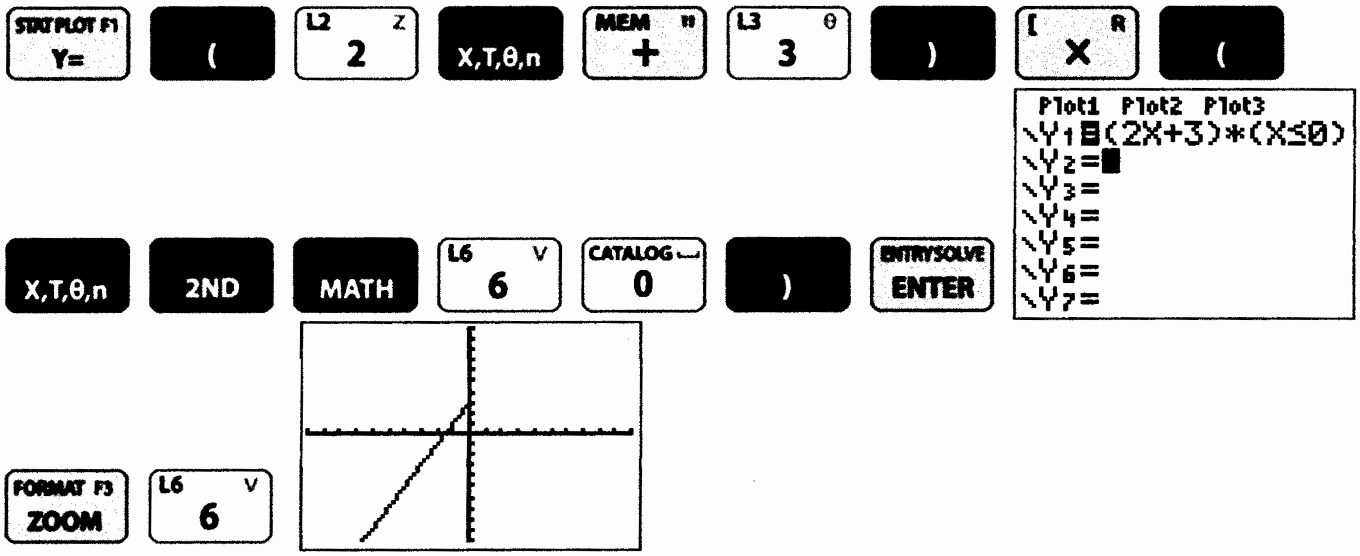
The GC graphs a horizontal line $y=0$ for all x values before $x=-2$, then a horizontal line $y=1$ for all x-values after -2.



The table shows that $x=-2$ is assigned y-coordinate 0.

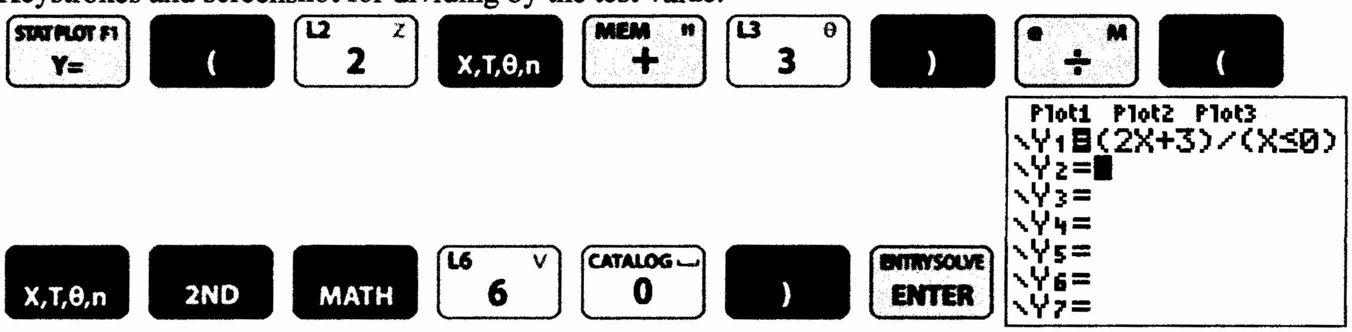
Example 4: Graph the function $f(x) = 2x + 3$ for the domain $x \le 0$ by multiplying by the TEST results for its domain.

CAUTION: We want the function value times the test value, so we need to include additional parentheses!



CAUTION: If you have a new color calculator, you may see a horizontal line $y = 0$ for the x-values on the right side of the graph. You can remove this by *DIVIDING* by the test value. Newer calculators do not graph values that result in $\div 0$.

Keystrokes and screenshot for dividing by the test value:

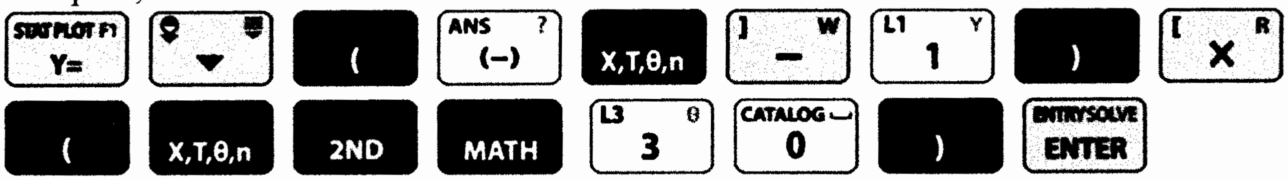


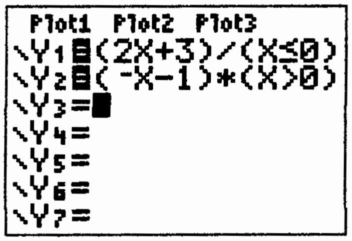
CAUTION: However, if you have an older calculator, dividing by the test value will completely confuse your calculator and it may graph nothing at all!

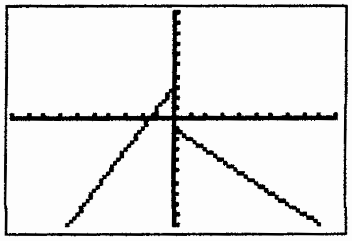
Example 5: Graph the piecewise function $f(x) = \begin{cases} 2x+3 & x \le 0 \\ -x-1 & x > 0 \end{cases}$ by multiplying each piece by TEST results for its domain using two lines of the y= menu.

The first piece is the same as Example 4. We continue with the second piece in y_2:

Example 5, continued





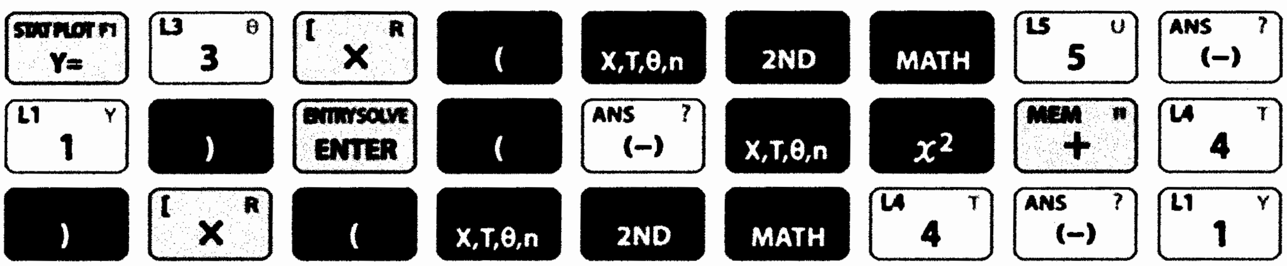


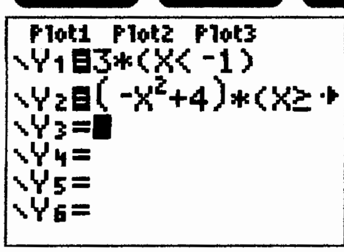
CAUTION: When you transfer this graph to paper, you must clearly indicate which piece includes its endpoint (closed circle) and which piece does not (open circle)!

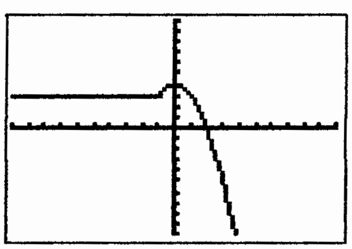
REMEMBER: A piecewise function is still a single function, and the resulting graph must pass the vertical line test!

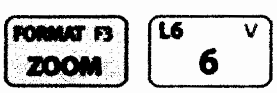
Example 6: Graph the piecewise function $f(x)=\begin{cases}3 & x<-1\\ -x^2+4 & x\geq -1\end{cases}$.

We expect the first piece to be a horizontal line while the second piece is a parabola.









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Practice: Graph the piecewise function.

1) Graph $f(x)=\begin{cases} x+2 & x<1 \\ 2x+1 & x\geq 1 \end{cases}$

2) Graph $f(x)=\begin{cases} -2x+4 & x\leq -1 \\ 3 & x>-1 \end{cases}$

3) Graph $f(x)=\begin{cases} x-1 & x\leq 3 \\ -x+5 & x>3 \end{cases}$

4) Graph $f(x)=\begin{cases} 4x-4 & x<2 \\ -x+1 & x\geq 2 \end{cases}$

5) Graph $f(x)=\begin{cases} -3x & x\leq -2 \\ 3x+2 & x>-2 \end{cases}$

6) Graph $f(x)=\begin{cases} -1 & x<-3 \\ 2 & x\geq -3 \end{cases}$

7) Graph $f(x)=\begin{cases} -x+7 & x<-1 \\ -x^2+9 & x\geq -1 \end{cases}$

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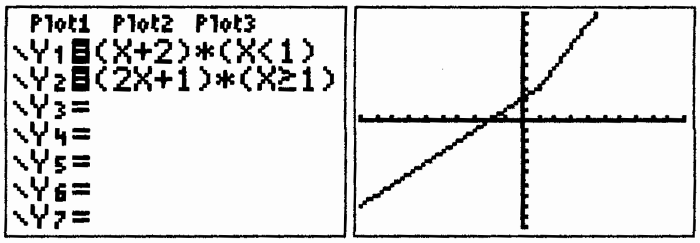
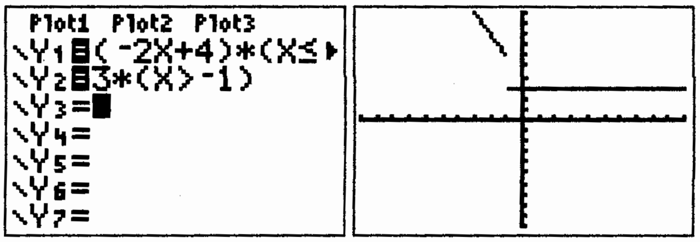
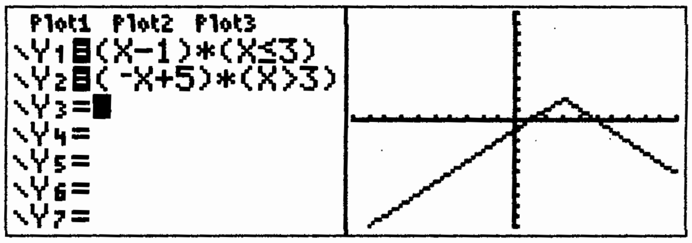
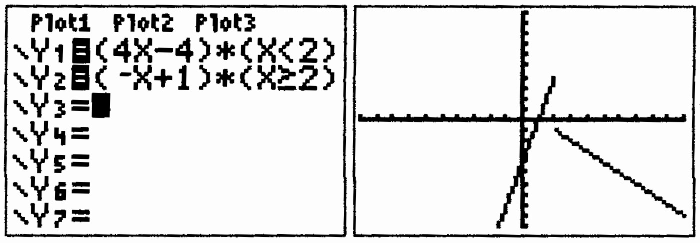
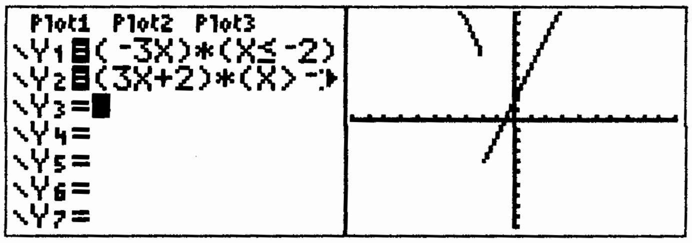
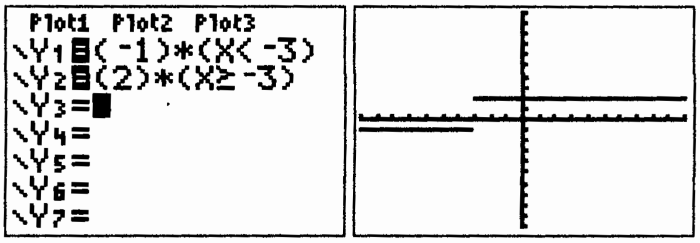
1) 

2) 

3) 

4) 

5) 

6) 

7)